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An investigation of dynamic behavior of the cylindrical shells under thermal effect

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ABSTRACT

Study the vibration characteristics of the cylindrical shell is considered a very important issue, because of the cylindrical shells are used for different applications in engineering fields such as missiles, electric motors, rocketry, etc. In different applications, the cylindrical shells have produced a high level of noise and vibration that effect on the behavior and performance of the systems. In some cases, when those systems work under these conditions for enough time, this will lead the systems to failure or at least change the situation of those systems from stable zone to unstable zone. In this paper, the finite element method was used to investigate deeply the vibration characteristics of cylindrical shells under different surrounding temperatures. Furthermore, the effect of thickness on the dynamic characteristics was investigated.

1. Introduction

A circular cylindrical shell is an essential element in different fields of engineering applications such as aircraft, locomotive, gas turbines and many other systems possess one piece or more of a circular cylindrical shell. The domain of vibration principles compact with the action of vibrating systems. The flexural vibration and fatigue fracture of a circular cylindrical shell working under the thermal effect becomes a very interesting subject in mechanical engineering design. The cylindrical shells may appear in different application such as missiles, storage tanks, and pressure vessels. All these applications require intensive study in both static and dynamic analysis.

Therefore, it's necessary to investigate the factors that affected the material properties (e.g. modulus of elasticity) and stiffness of the system and eventually how these factors will effect on the dynamic characteristics of the system. On the other hand, find out the effect of changing in the material properties and the stiffness on the behavior and performance of the system. Most of the studies about the cylindrical shell relevant to find the effect of dimensions (thickness and length) on the natural frequencies, but there are very seldom researchers studied the thermal effects on the vibration characteristics of the cylindrical shell.

Ahmadian and Bonakdar [1] applied the finite element method to find the solutions of the static and modal problems of laminated hollow cylinders. The super-element was used in their analysis consist of 16 nodes and each node has 6 degrees of freedom. Different external loads were subjected to assuming different working conditions. The results proved that the model was reliable and the percentage error was acceptable.

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Li and Lam [2] investigated the frequency characteristics of the rotating cylinder shell using the generalized differential quadrature method (GDQ). Their studies based on Love-type shell theory to transform the three dimensional dynamic problems to one dimensional which is the meridional direction. Furthermore, the influences of rotating speed on the correlation between the wave number in the circumferential direction and the frequency; their results had a good agreement with other researchers that used different methods.

Loy and Lam [3] investigated the impacts of the tension which occurred at the initial time, centrifugal force and Coriolis on the dynamic behavior of the thin cylindrical panels (rotating). The Eigen-solutions are obtained by using the Newton-Raphson method to carry out their analysis. They found that the behaviors of natural frequencies of thin cylindrical panels (rotating) are like those which appeared in the rotating cylinder shell. The results proved that the proposed approach was valid.

Guo et al. [4] used nine-nodes super-parametric element to build the finite element model in order to study the vibration of rotating cylindrical shells, they took into consideration the effects of initial tension and its effect on the nonlinear geometry because of large deformation Coriolis acceleration, Centrifugal force. Also, they investigated the effect of rotation in three dimensions modes of the cylindrical shell.

Popov [5] presented the developed model to explain how the dynamic loads affect the buckling and the vibration status of the shells under parametric excitation assuming the bifurcation and dynamics nonlinearity. He spotted light on the nonlinearities which existed in geometry that affect the vibrations of circular cylindrical shells and he concluded that the linear theory will not suitable to obtain accurate results when the amplitude becomes comparable to the thickness of the shell.

Ng and Lam [6] studied the effects of constant axial loads on the vibration and critical speed of thin isotropic cylindrical shells applying Donnell's theory. The effects of both of the centrifugal and Coriolis forces were taken into consideration.

Tafreshi [7] formulated a model to compute the delamination in isotropic and laminated composite cylindrical shells using the numerical technique (finite element method) with combined of single and double layers. The effects of the delamination size, orientation and through-the-width position with a series of laminated cylinders were investigated. The change in properties of the materials was taken into consideration in the mathematical model.

Omer Civalek [8] built different mathematical models of rotating shells (conical shape) working in different working conditions and then applied the discrete singular convolution method to analyze the vibration characteristics of these shells.

Saito and Endo [9] applied the equations of the Flügge's to find the vibration characteristics of the finite length of rotating cylinder shell. They took into consideration the initial tensions which occurred due to the rotation in the cylinder shell. Three types of boundary conditions were assumed using Galerkin's method to analyze the frequencies during the traveling waves. The boundary conditions included the full clamped, the simply supported without axial constraint and the simply supported with axial constraint. The conclusions of this study proved that the frequencies were a function of the rotational speed and considerably affected by the restriction conditions.

Xuebin [10] developed a method based on Flügge's shell theory equations to calculate the free vibration frequencies of the thin circular cylindrical shell assuming the materials were orthotropic. This work focused on the coupled polynomial eigenvalue problem and the results had a good agreement based on the comparison between the classical dynamics approach and the developed one.

Leu [11] studied the effect of strain-hardening visco-plastic on the dynamic behavior of the rotating hollow cylinders using the sequential limit approach. The analytical solutions of the dynamic problem included angular speeds (less than a certain value) were derived. The results of the rigorous which located upper bounds agreed with cases of the limit of angular speeds that calculated using the analytical solution.

Liew et al. [12] explored the stability status according to the dynamic issue of the rotating cylindrical shells under the steady-state and when applied the periodic axial forces using a combination of Bolotin's (1st approximation) and Ritz methods. The solutions of the system of equations (Mathieu–Hill) were presented based on Ritz energy to minimize magnitude. The work included the effects of the hoop tension and Coriolis effects due to the rotation.

The buckling and free vibration analyses of the laminated hat-stiffened shallow were investigated by Prusty [13] using finite element method. The solutions were found for the different design of the stiffeners such as 'T' section and hat shape. The results approved that the stiffened panels showed superior performance compared with the open section stiffeners.

The objective of this research paper is to investigate the dynamic characteristics of any structure under different boundary and load conditions using finite element technique. Where there are many researchers investigated the effect of rotating and damage on the dynamic behaviors of structures [14–18], but a few researchers investigated the thermal effect.

In this work, the mathematical model of a cylindrical shell was developed taking into account the thermal effect of the surrounding on the dynamic characteristics. The general finite element program has been built using Fortran90 Language to compute the frequencies and mode shapes of the cylindrical shell under different boundary condition. A wide range of surrounding temperature (between − 200 °C and 200 °C) was assumed to investigate the change in the natural frequencies of the cylindrical shell due to the thermal effect.

2. Finite element formulations

The superparametric shell element was selected to build the finite element model of the cylindrical shell. FORTRAN90 has been used to build a specific program to investigate the vibration characteristics of the cylindrical shell. The selected element consists of eight nodes with 5-degrees of freedom for each node. The development and applications of isoparametric elements family were presented by Zienkiewicz [19]. This section presents the equations which used to find the stiffness and mass matrices of the superparametric shell element. General displacements vector at any node in the structure using shell element according to the global

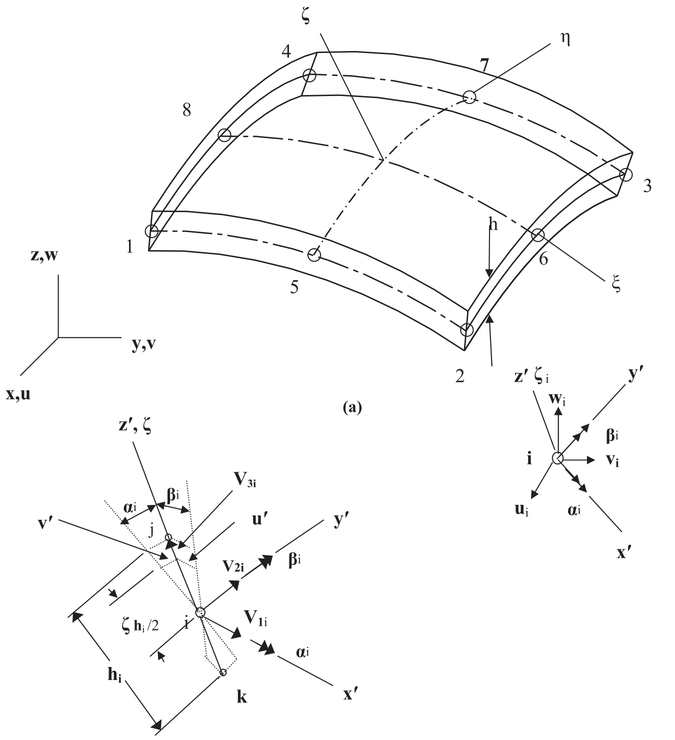

Fig. 1. (a) Shell element with 8-node (b) Nodal vector.

axis are:

$$U = \{u, v, w\} \quad (1)$$

The rotations about the local tangential axes $X^{`}$ and $Y^{`}$ are α and β as shown in Fig. 1(a) hence:

$$qi = \{u_i, v_i, w_i, \alpha_i, \beta_i\} \quad (i = 1,\ 2.8) \quad (2)$$

In the kinematics formulation, there are two assumptions are imposed: the nodal fiber is inextensible and there is only small rotations were considered. The general nodal displacements are,

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{i=1}^{8} N_i \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} + \sum_{i=1}^{8} N_i \zeta \frac{h_i}{2} \mu_i \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \quad (3)$$

where N_i is the shape function and μ_i denotes the following Matrix:

$$\mu_i = \begin{bmatrix} -l_{2i} & l_{1i} \\ -m_{2i} & m_{1i} \\ -n_{2i} & n_{1i} \end{bmatrix} \quad (4)$$

Fig. 1(b) shows the 1st tangential vector V_{1i} and the 2nd tangential vector V_{2i}. All values of the 1st column are negative and represent the direction cosines of 2nd tangential vector V_{2i}. While all values of the 2nd column are positive values and represent the 1st tangential vector V_{1i}. These vectors (V_{1i} and V_{2i}) are orthogonal to the vector V_{3i}, and also to each other. The displacements in the global coordinates are

$$\begin{bmatrix} u_{,x} \\ u_{,y} \\ u_{,z} \\ v_{,x} \\ v_{,y} \\ v_{,z} \\ w_{,x} \\ w_{,y} \\ w_{,z} \end{bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} a_i & 0 & 0 & -d_i l_{2i} & d_i l_{1i} \\ b_i & 0 & 0 & -e_i l_{2i} & e_i l_{1i} \\ c_i & 0 & 0 & -g_i l_{2i} & g_i l_{1i} \\ 0 & a_i & 0 & -d_i m_{2i} & d_i m_{1i} \\ 0 & b_i & 0 & -e_i m_{2i} & e_i m_{1i} \\ 0 & c_i & 0 & -g_i m_{2i} & g_i m_{1i} \\ 0 & 0 & a_i & -d_i n_{2i} & d_i n_{1i} \\ 0 & 0 & b_i & -e_i n_{2i} & e_i n_{1i} \\ 0 & 0 & c_i & -g_i n_{2i} & g_i n_{1i} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{bmatrix} \tag{5}$$

where

$$a_i = J^*{}_{11} N_{i,\zeta} + J^*_{12} N_{i,\eta} \qquad d_i = \frac{h_i}{2}(a_i \zeta + J^*_{13} N_i)$$
$$b_i = J^*_{21} N_{1,\zeta} + J^*_{22} N_{i,\eta} \qquad e_i = \frac{h_i}{2}(b_i \zeta + J^*_{23} N_i)$$
$$c_i = J^*_{31} N_{i,\zeta} + J^*_{32} N_{i,\eta} \qquad g_i = \frac{h_i}{2}(c_i \zeta + J^*_{33} N_i)$$

It can be obtained the stiffness matrix from the following form,

$$[K] = \int_{vol} [B]^T [D][B] \; dV \tag{6}$$

where

$$B_i = \begin{bmatrix} a_i & 0 & 0 & -d_i l_{2i} & d_i l_{1i} \\ 0 & b_i & 0 & -e_i m_{2i} & e_i m_{1i} \\ 0 & 0 & c_i & -g_i n_{2i} & g_i n_{1i} \\ b_i & a_i & 0 & -e_i l_{2i} - d_i m_{2i} & e_i l_{1i} + d_i m_{1i} \\ 0 & c_i & b_i & -g_i m_{2i} - e_i n_{2i} & g_i m_{1i} + e_i n_{1i} \\ c_i & 0 & a_i & -d_i n_{2i} - g_i l_{2i} & d_i n_{1i} + g_i l_{1i} \end{bmatrix} (i = 1, 2, \ldots, 8) \tag{7}$$

and $[D]$ is the rigidity matrix. A typical rigidity matrix is given by,

$$[D] = \frac{E(T)\,h}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2k_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2k_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{h^2}{12} & \frac{h^2\nu}{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{h^2\nu}{12} & \frac{h^2}{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h^2(1-\nu)}{24} \end{bmatrix}$$

where E is Young modulus and its function of temperature $[E = f(T)]$ and υ is Poisson's ratio. k_1 is the shear correction factor (assumed $k_1 = 1.2$ [20]). The mass matrix form is,

$$[M] = \int_V \rho [N]^T [N] \, dV \tag{8}$$

3. Vibration analysis

The general equation of motion to find the vibrational characteristics of any structure is [20]:

$$[M]\{\ddot{U}\} + [K]\{U\} = 0 \tag{9}$$

where

$$U = \Phi_i \sin(\omega_i \, t + \theta_i), \qquad i = 1, \; 2, \; \ldots.DOF \tag{10}$$

where Φ_i is a vector of mode shape of i^t mode of vibration, ω_i is the angular frequency (mode i) and θ_i is the phase angle. After differentiating Eq. (10) twice with respect to time t, it can get the following:

$$\ddot{U}_i = -\omega_i^2 \Phi_i \sin(\omega_i t + \theta_i) \tag{11}$$

Substitution of Eqs. (11) and (10) into Eq. (9) and allows cancellation of the term sin $(\omega_i \text{ t} + \theta_i)$ yield the following equation,

$$([K] - \omega_i^2[M])\Phi_i = 0 \tag{12}$$

Eq. (12) has the form of the algebraic eigenvalue problem. The most efficient type of Eq. (12) for the structural vibrations accepts of the eigenvalue problem only in the following standard symmetric form:

$$([A] - \lambda_i[I])XX_i = 0 \tag{13}$$

In which $[A]$ is a symmetric matrix (dynamic matrix) and $[I]$ is an identity matrix. The symbol λ_i denotes the ith eigenvalue. XX_i is the corresponding eigenvector of the new system where the equations are homogeneous. Putting Eq. (12) into the form of Eq. (13) by factoring either matrix $[K]$ or matrix $[M]$, using the Cholesky square root method (which is a direct method for solving a linear system based on the fact that any square matrix $[A]$ can be expressed as the product of an upper and lower triangular matrix [20]). The steps to solve Eq. (12) using Cholesky square root method (if the stiffness matrix is positive definite) are:

- Choose of factor $[K]$ for an important reason that will soon be apparent. Thus:

  $$[K] = [Q]^T [Q] \tag{14}$$

  where the factor $[Q]$ is an upper triangular matrix.
- Substitute Eq. (14) into Eq. (12) to obtain:

  $$([Q]^T[Q] - \omega_i^2[M])\Phi_i = 0 \tag{15}$$

- Pre-multiply Eq. (15) by $[Q]^{-T}$ and insert $I = [Q]^{-T}[Q]$ after matrix $[M]$, which yields:

  $$[Q]^{-T}([Q]^T[Q] - \omega_i^2[M])\Phi_i = 0 \tag{16}$$

- Rearrange the terms in reverse order, then it can be found the following form:

  $$([M_A] - \lambda_i[I])\Phi_{Ai} = 0 \tag{17}$$

  where,

  $$[M_A] = [Q]^{-T}[M][Q]^{-1} \tag{18}$$

  and

  $$\lambda_i = \frac{1}{\omega_i^2}\Phi_{Ai} = [Q]\ \Phi_i \tag{19}$$

- The angular frequencies and mode shapes in the original coordinate can be determined as:

  $$\omega_i = \frac{1}{\sqrt{\lambda_i}}; \quad \Phi_i = [Q]^{-1}\Phi_{Ai} \tag{20}$$

Because the matrix M_A in the new coordinates is symmetric, all of its eigenvectors are linearly independent.

4. Free Vibration Analysis Program (FVAP)

In order to analyze the dynamic characteristics of cylinder shell a finite element program was coded using Fortran90 language. The block diagram of the Free Vibration Analysis Program (FVAP) is shown in Fig. 2. The aim of this program is to analyze the vibration characteristic of any structure in general, but the AutoMesh in this program is built specifically for the cylinder shell. Also, it can be update AutoMesh in the future depend on the select case study. The details of the "Free Vibration Analysis Program" as follows:

- FVAP: is the master program to control all existing subroutines and also opening the files to read the inputs and the print output files.
- Data: the task of this subroutine is to accept the data from the input files, which is open from master routine. The data of the output file contain two subroutines:
  - General: this subroutine is used to define the structural outline of the cylindrical shell.
  - AutoMesh: this subroutine divides the structure into elements and nodes.
- Assemb: the task of this subroutine is to assemble and calculate the model stiffness matrix and mass matrix and its containing of 4 subroutines:
  - Bmatrix: this subroutine is used to evaluate the strain matrix (B-matrix) and transformation of B to the local coordinates.

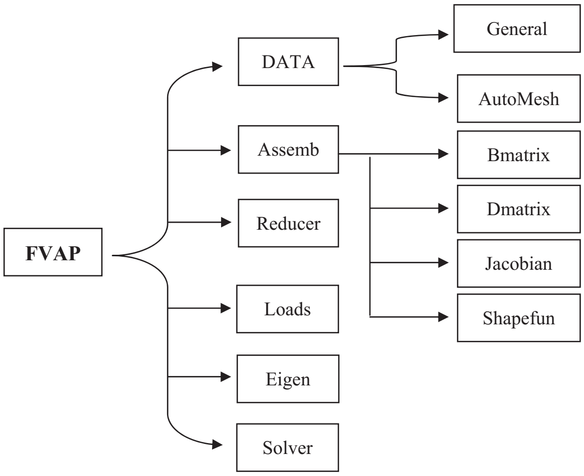

Fig. 2. Block diagram of Free Vibration Analysis Program (FVAP).

- Dmatrix: this subroutine is used to calculate the matrix of elastic rigidities.
- Jacobian: this subroutine is used to compute the Jacobian matrix, derivatives, determinant and inverse.
- Shapefun: this subroutine is used to calculate the shape function and derivatives.

● Eigen: this subroutine is used to calculate and solve the Eigenvalue, Eigenvector problems and print it in the output files.
● Solver: This subroutine is used to solve the system of equations using Cholesky's method.

The convergence tests were made for each case to select the suitable mesh size of the final model of the cylindrical shell in order to guarantee the accuracy of the results (Fig. 3). The material is assumed isotropic and there is no damage or crack in the structure of the cylindrical shell. The material properties and dimensions of the cylindrical shell are listed in Table 1.

5. Results and discussions

In this paper a series of computations were conducted to investigate deeply the effect of surrounding temperature on the dynamic behavior of the cylindrical shell. This analysis is completed using three-dimensional models to simulate the vibration of the cylindrical shell using different thicknesses. Six modes (n,m) [(0,2) (2,4), (0,3), (2,6), (0,4) and (2,8)] were presented in the results, where m is the axial mode and n is the circumferential mode.

Figs. 4–6 demonstrate the variation of natural frequencies of the cylindrical shell under different surrounding temperature using different thicknesses (0.75 mm, 1 mm, 1.5 mm, 3 mm and 4.5 mm). A wide range of surrounding temperature was assumed in this analysis which started from $-200\,^{\circ}C$ to $200\,^{\circ}C$ in order to understand deeply the dynamic characteristics of the cylindrical shell under different thermal conditions. It's observed that the values of natural frequencies of all cases decreased when the surrounding temperature increased; because of the material properties is a function of the temperature. Generally, the stiffness of any structure is a

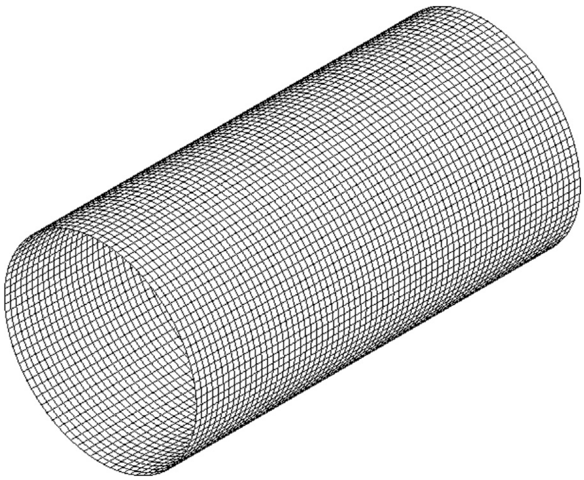

Fig. 3. Finite element model of the cylindrical shell (No. of elements is 6300).

Table 1
Material properties of Aluminum Alloy (3003, 3004 & 6063) and dimensions of the cylindrical shell.

Parameters	Values	
Modulus of Elasticity [GPa]	76.5	− 200 °C
	72.7	− 100 °C
	68.9	25 °C
	65.8	100 °C
	62.7	150 °C
	57.7	200 °C
Density [kg/m^3]	> 2740	
Poisson's Ratio	0.33	
Length [m]	0.6	
Radius [m]	0.15	

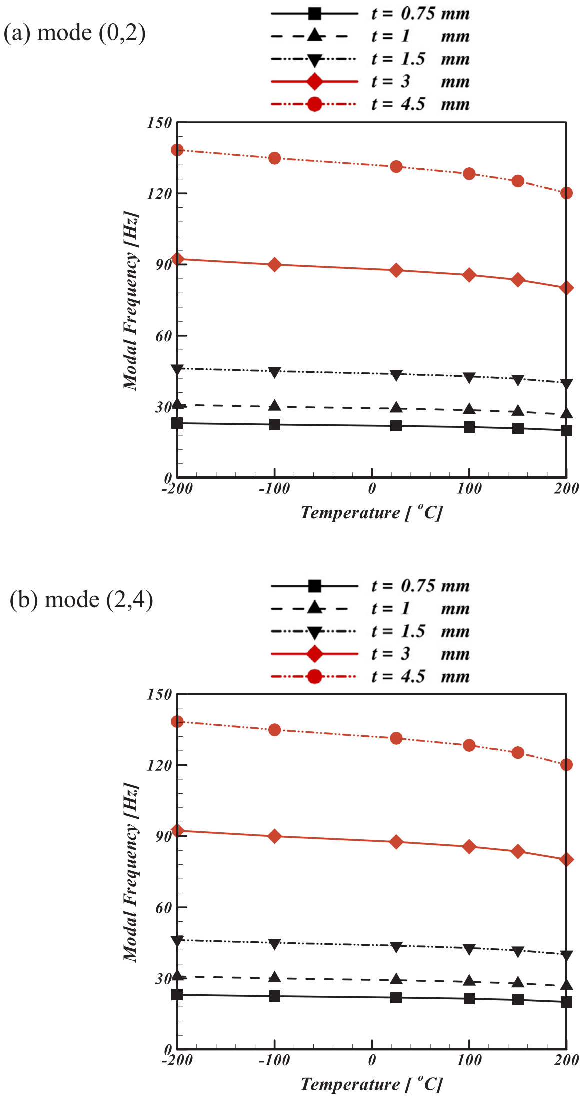

Fig. 4. Modal frequencies (Hz) of the cylindrical shell.

function of the material properties (e.g. Modulus of Elasticity), therefore any factor affecting the material properties such as the high temperature will lead to change the stiffness of the structure and eventually affect dramatically the dynamic characteristics of the structure. When the surrounding temperatures change from − 200 °C to 200 °C and from 25 °C to 200 °C the percentages reduction in

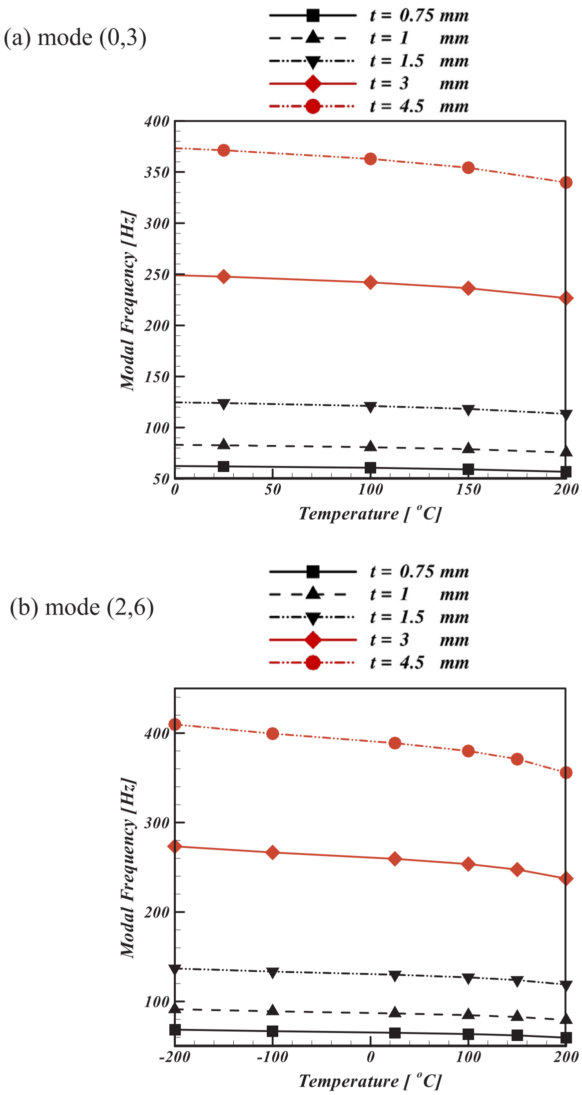

Fig. 5. Modal frequencies (Hz) of the cylindrical shell.

the natural frequencies are found approximately 13% and 8.5% for all modes, respectively.

It can be noticed that when the thickness of cylindrical shell increases the natural frequencies increase too. The percentage increases in the natural frequencies for all modes are found to be approximately 498% when thickness change from 0.75 mm to 4.5 mm, except for the mode (0,2) is found 349%. One of the significant factors affected the stiffness of any structure is the thickness, therefore when increasing the thickness of the cylinder shell the stiffness increases too. The increment in the stiffness of cylindrical shell led to increase the values of the natural frequencies dramatically.

6. Conclusions and remarks

The finite element program was built based on the developed theoretical model to investigate the influence of the surrounding temperature on the vibration characteristics of a circular cylindrical shell. Three-dimensional models were used to simulate the dynamic response of the cylindrical shell. The program was coded using Fortran language to study the vibrational characteristics of the cylindrical shell. The results showed that the Fortran program provides excellent approximations to find the natural frequencies and mode shapes of the cylindrical shell.

The increment in the surrounding temperature lead to decrease the magnitude of Young's modulus, and this effect will be more significant in the high temperature range. The values of the natural frequencies of all cases decreased due to the increases in the surrounding temperatures. The reason for these results is the reduction in the stiffness of structure. Also, the natural frequencies increased when the thickness of the cylindrical shell increased due to the increase in the structural stiffness.

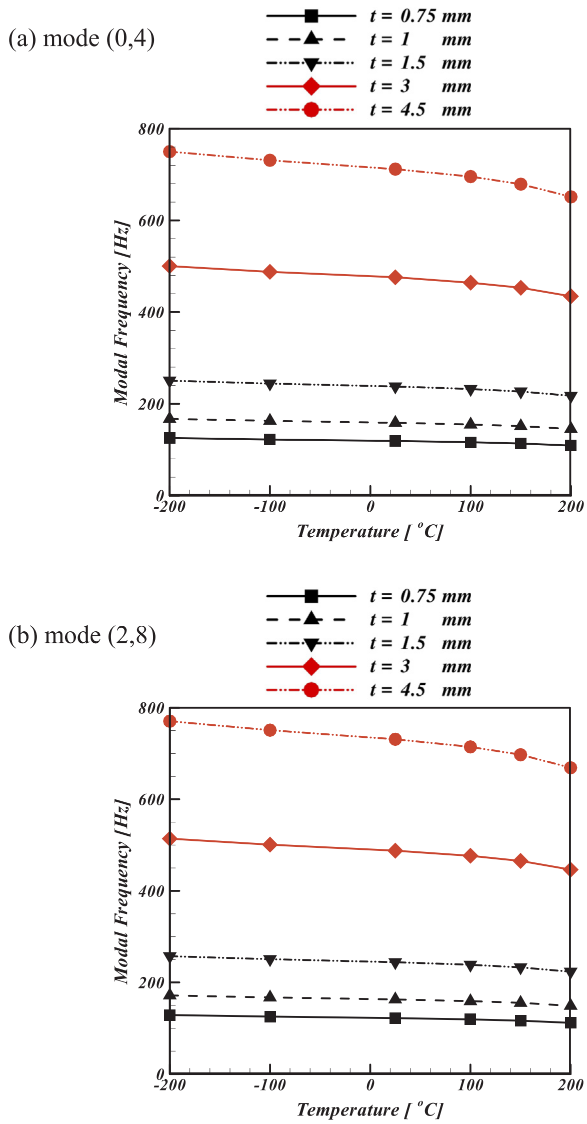

Fig. 6. Modal frequencies (Hz) of the cylindrical shell.

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